Adaptive robust disturbance compensating control for a servo system in the presence of both friction and deadzone

Jinho Jung¹, Donghyuk Lee², Jong Shik Kim¹ and Seong Ik Han²

Abstract
An adaptive robust control that does not need sophisticated plant modeling work is proposed for precise output positioning of a servo system in the presence of both friction and deadzone nonlinearities. It is difficult to achieve effective motion control by traditional linear control methodology for these types of nonlinearities, without the aid of a proper compensation scheme for nonlinearity. In this study, dynamic friction is modeled by a Tustin friction model, and inverse deadzone method is adopted to compensate deadzone effect. The adaptive laws of the unknown system dynamic parameters, friction and deadzone, are derived. Furthermore, a robust control method with funnel control is proposed to compensate for unmodeled and estimation errors. The boundedness and convergence of the closed-loop system are ensured by a Lyapunov stability analysis. The performance of the proposed control scheme is verified through experiments on the XY table servo system and the robotic manipulator.

Keywords
Adaptive control, deadzone, friction, funnel control, robust uncertainty compensator, servo system

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Introduction
In recent times, with the increasing demand for high quality production, precise motion in industrial servo machines, such as machine tools and industrial robots, has become an important requirement. A wide variety of control strategies have been investigated for mechanical servo systems including PD and nonlinear feedback control ¹² and the open-loop optimization control.³⁴ In most servo machines, besides mechanical elements such as inertia, and coupled dynamics, there are often inherent nonlinear elements such as friction and deadzone in the contact surface and in the transmitting power actuator; it is often difficult to compensate for these parameters by a simple linear controller. Nonlinear friction gives rise to transmission lag between contact surfaces, and deadzone leads to loss of power efficiency. In order to achieve both high motion control performance and convenient applicability of the servo system, a reasonable identification and modeling process for these mechanical parameters, and efficient control algorithm without depending on a complex control structure are prerequisites.

Nonlinear friction models include classic Tustin friction model⁷ and dynamic friction models such as the LuGre model⁶ and generalized Maxwell-slip friction model (GMS).⁷ The classic friction model cannot compensate for pre-sliding friction properties. While the dynamic friction model operates in the pre-sliding friction, the shortcomings of these advanced friction models are that there are a lot of parameters which lead to failures in real-time operation, and require a deep understanding of the machine structure system. Several researchers have developed deadzone compensation methods in the servo and robotic systems.⁸⁻¹⁰ In most cases, modeling of deadzone is not simple because of irregular and asymmetric width and slope of the deadzone. The approaches for the servo system control that take case of both friction and deadzone are rare; the limitations are on account of the difficulty in obtaining exact compensation for friction and deadzone. Some control approaches for servo mechanical systems without modeling,...
mechanical parameters are found in the literatures\textsuperscript{11–14} wherein the adaptive estimation-based control was employed instead of the model-based control. These adaptive estimation-based controls showed efficient control performance to some extent in applications to robotic manipulator systems without the presence of friction or deadzone, or both.

In this paper, the adaptive laws were developed not only for the unknown parameters such as inertia, coupled dynamics, and gravity term, but also for friction and deadzone parameters in order to design a non-model-based controller without introducing any modeling procedure, for ensuring fast controller design and improved tracking motion in industrial applications. However, the adaptive estimation for unknown parameters does not always guarantee satisfactory performance because of its incomplete choice of the adaptive gain and difficulty in determining the optimal adaptive gain. Generally, it is known that the switching law compensates for unknown estimation errors and disturbances in sliding mode control (SMC).\textsuperscript{15} However, this control causes chattering, generated by the sign function and high frequency oscillation in the control action. Therefore, a robust disturbance compensation law that borrowed its concept from the funnel control\textsuperscript{16,17} is considered to add robustness to the adaptive controller without generating chattering during tracking. The gains of the funnel-based compensator are obtained from the estimate for the unknown upper bound of disturbance. From the Lyapunov function, all the adaptive laws are derived, and a stability analysis is performed. The designed controller is implemented using Matlab Realtime Toolbox and MF624 interface board\textsuperscript{18} for conducting experiments on the XY table and the articulate robot manipulator.

\textbf{Problem formulation}

\textbf{Description of a non-smooth nonlinear servo system}

Consider a servo system in the presence of both deadzone and friction, which is described by the dynamic equation

\[ M(q) + C(q, \dot{q}) + G(q) + F_f(q, \dot{q}) + F_d(t) = D(u) \]

where \( q = [q_1, \ldots, q_n]^T \in \mathbb{R}^n \) denotes the vector of generalized coordinates; \( M(q) \in \mathbb{R}^{n \times n} \) represents the symmetrically bounded positive definite matrix; \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the centripetal Coriolis torque matrix; \( G(q) \) is the gravity force vector; \( F_f(q, \dot{q}) \in \mathbb{R}^n \) is the nonlinear friction vector; \( F_d \in \mathbb{R}^n \) is the external disturbance; and \( u \in \mathbb{R}^n \) is the control input vector.

\textbf{Property 1.} The inertia matrix, \( M(q) \), is known to be symmetric and positive definite.

\textbf{Property 2\textsuperscript{19}.} Since \( M(q) \) and therefore \( M(q) \) are symmetric matrices, the skew-symmetry of the matrix \( M(q) - 2C(q, \dot{q}) \) can be also be seen from the fact \( M(q) = C(q, \dot{q}) + C^T(q, \dot{q}) \).

\textbf{Property 3\textsuperscript{19}.} Even though the skew-symmetry property of \( M(q) - 2C(q, \dot{q}) \) is guaranteed if \( C(q, \dot{q}) \) is defined by the Christoffel symbols, it is always true that \( \dot{q}^T[M(q) - 2C(q, \dot{q})]q = 0 \).

\textbf{Assumption 1.} There exist some finite positive constants, \( k_i > 0, 1 \leq i \leq 4 \) and finite nonnegative constants, such that \( \forall q \in \mathbb{R}^n, \forall \dot{q} \in \mathbb{R}^n, \ |M(q)| \leq k_1, |C(q, \dot{q})| \leq k_2, |G(q)| \leq k_3 \), and \( \sup_{t \geq 0} |F_d| \leq k_4 \).

The deadzone nonlinearity \( D(u) \) is shown in Figure 1(a) and a mathematical model is described by

\[ D(u) = \left\{ \begin{array}{ll} m_r(u(t) - d_r) & \text{for } u(t) \geq d_r \\ 0 & \text{for } d_r < u(t) < d_l \\ m_l(u(t) - d_l) & \text{for } u(t) \leq d_l \end{array} \right. \]

where \( m_r \) and \( m_l \) denote the slope of the deadzone, and \( d_r \) and \( d_l \) stand for the deadzone width parameters.

The practical assumptions concerning the deadzone are given below for the control problem:

\textbf{Assumption 2.} The deadzone output is not available for measurement. The deadzone parameters, \( d_r \) and \( d_l \), are unknown but their signs are known as \( d_r \geq 0 \) and \( d_l \leq 0 \).

\textbf{Assumption 3.} The deadzone slopes are assumed as \( m_r = 1 \) and \( m_l = 1 \) to simplify the problem.

The deadzone inverse technique is a useful method for compensating the deadzone effect.\textsuperscript{8,9,10} Setting \( u_d(t) \) as the control signal from the controller for achieving control of the plant without deadzone, the following control signal \( u(t) \) is generated according to certainty equivalence deadzone inverse described in Figure 1(b)

\[ u(t) = D^{-1}(u_d(t)) = (u_d(t) + \hat{d_r})p + (u_d(t) + \hat{d_l})(1 - p) \]

where \( \hat{d_r} \) and \( \hat{d_l} \) are the estimates of \( d_r \) and \( d_l \), respectively and

\[ p = \begin{cases} 1 & \text{if } u_d(t) \geq 0 \\ 0 & \text{if } u_d(t) < 0 \end{cases} \]

The resulting error between \( u \) and \( u_d \) is given by

\[ D(u(t)) - u_d(t) = \hat{d_r}p + \hat{d_l}(1 - p) \]

The resulting control system with deadzone inverse is shown in Figure 2.
The friction force as the classical Tustin model can be expressed as

\[ F_f = F_c + (F_s - F_c) \exp\left(-\left(\frac{|v|}{v_s}\right)^2\right) \text{sgn}(v) + F_v v \]

where \( F_c \) denotes Coulomb friction, \( F_s \) denotes the stiction level, \( F_v \) represents the viscous friction, and \( v_s \) is the Stribeck velocity. The estimation of the friction force can be obtained from

\[ \hat{F}_f = \hat{F}_c + (\hat{F}_s - \hat{F}_c) \exp\left(-\left(\frac{|v|}{v_s}\right)^2\right) \text{sgn}(v) + \hat{F}_v v \]

where \( \hat{F}_i \) are estimates of \( F_i \), \( i = f, c, s, v \), respectively. From equations (6) and (7), the following expression is obtained

\[ \tilde{F}_f = \hat{F}_f - \hat{F}_f \]

\[ = \hat{F}_c \left[ 1 - \exp\left(-\left(\frac{|v|}{v_s}\right)^2\right) \right] \text{sgn}(v) + \hat{F}_v v \]

where \( \tilde{F}_f = F_f - \hat{F}_f \).

Design of controller and nonlinearity compensator

In this section, three controllers are designed and the adaptive laws for the unknown system parameters are derived from the stability analysis, by using Lyapunov function. First, a model reference controller is designed under the assumption that the dynamic parameters of the servo system are known. However, this controller cannot be applied efficiently to a real system because most of the dynamic parameters are unknown. The second adaptive controller is then designed using estimated parameters from the adaptive laws for the unknown dynamic parameters. In the third step, since the adaptive laws cannot estimate the unknown parameters precisely and some estimate errors exist, a robust control is added in order to ensure robustness of the adaptive controller, by introducing funnel control, without considering relay control of the conventional SMC, which generates undesirable chattering.

Model reference control

The control objective for a servo dynamic system is to determine a state feedback control system such that the system output \( q \) can track a desired trajectory \( q_d \), while ensuring that all the closed loop signals are bounded. Consider the following signals

\[ e = q - q_d \]

\[ \dot{r} = \dot{q}_d - \zeta e \]

where \( q_d = [q_{d1}, \ldots, q_{dn}]^T \) is the desired trajectory, \( \zeta = \text{diag}(\zeta_1, \ldots, \zeta_n) \) is the constant matrix, and \( e = [e_1, \ldots, e_n]^T \). The filtered error surface \( s \) and its derivative \( \dot{s} \) are defined as

\[ s = \dot{e} + \zeta e \]
\[
\begin{align*}
\dot{s} &= \ddot{q} + \zeta \varepsilon \\
\dot{\varepsilon} &= \ddot{\zeta} + \dot{\zeta} \varepsilon \\
\dot{\zeta} &= \ddot{\zeta} + \zeta \varepsilon
\end{align*}
\]

\[\text{Based on the terms of the definitions (9) to (12) and if the deadzone is compensated by the inverse deadzone technique precisely, i.e. } D(u(t)) = u_d(t) \text{ in (5), the dynamic equation (1) can be written as}
\]
\[
M(q)\ddot{s} = M(q)\ddot{q} - M(q)\dot{\dot{r}} - F_d(t) + u_d
\]

\[\text{Define the Lyapunov function candidate as follows}
\]
\[
V_1 = \frac{1}{2} s^T M(q)s
\]

\[\text{Considering (13) and the obtained result of } s^T [M(q) - 2C(q, \dot{q})] s = 0 \text{ from the property 3, the time derivative of } V_1 \text{ is written as}
\]
\[
\dot{V}_1 = s^T M(q) \ddot{s} + \frac{1}{2} s^T \dot{M}(q)s
\]

\[\text{where} \quad u = u_{eq} + u_s
\]

\[= -\zeta s - \Lambda_m \dot{\phi}_m - \beta \text{sign}(s)
\]

\[\text{Based on equation (16), equation (15) is written as}
\]
\[
\dot{V}_1 \leq -\zeta \|s\|^2 - \beta \|s\|^3
\]

\[\text{Integrating equation (17) leads to}
\]
\[
V_1(t) - V_1(0) \leq -\zeta \min \int_0^t \|s\|^2 dt \leq 0
\]

\[\text{Therefore, } s \text{ remains to small compact set containing the origin, as time goes to infinity. However, this model-based control requires exact information on mechanical dynamics and disturbances. When the dynamics are complex and disturbances are unknown, this control scheme undergoes limitations in real applications. Therefore, we consider an adaptive control method by adopting estimates of unknown dynamics and disturbances.}
\]

\[\text{Adaptive control}
\]

\[\text{In this section, the controller and adaptive laws are derived. Defining the tracking error as } e = q - q_d, \text{ the command vector, } r(t), \text{ and its derivative, } \dot{r}(t), \text{ are defined as}
\]
\[
r = \dot{q}_d - \zeta e
\]

\[\dot{r} = \ddot{q}_d - \zeta \dot{e}
\]

\[\text{Based on equations (5) and (8), and the terms of the definitions (19) and (20), the dynamic equation (1) can be written as}
\]
\[
M(q)\ddot{s} = -C(q, \dot{q})s - M(q)\dot{\dot{r}} - C(q, \dot{q})\dot{r} - G(q) - F_d(t) + u_d
\]

\[\text{as estimates of } k_i, \text{ and } \hat{k}_i = k_i - \hat{k}_i
\]

\[\text{As the dynamic parameters } k_i \text{ cannot be known a priori, adaptive laws are considered to estimate the unknown parameters. The Lyapunov function candidate is defined as follows}
\]
\[
V_2 = \frac{1}{2} s^T M(q)s + \sum_{i=1}^4 \frac{1}{2\eta_i} k_i \dot{k}_i + \sum_{j=1}^c \frac{1}{2\eta_j} \dot{F}_j F_j
\]

\[\text{where } \eta_i > 0, i = 1, \ldots, 4, \text{ and } \eta_j > 0, j = c, s, v, \text{ are constants. Differentiating equation (22) with respect to time, we obtain}
\]
\[
\dot{V}_2 = -s^T C(q, \dot{q}) - \frac{1}{2} \dot{M}(q)s + s^T \left[ -\dot{k}_2 \dot{r} - k_3 \right]
\]

\[\text{where } \eta_i > 0, i = 1, \ldots, 4, \text{ and } \eta_j > 0, j = c, s, v, \text{ are constants. Differentiating equation (22) with respect to time, we obtain}
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\]
\[
\dot{V}_2 = -s^T C(q, \dot{q}) - \frac{1}{2} \dot{M}(q)s + s^T \left[ -\dot{k}_2 \dot{r} - k_3 \right]
\]
where \( \hat{K} = [\hat{k}_1 \hat{k}_2 \hat{k}_3 \hat{k}_4]^T \) and \( \phi = [\|\tilde{r}\| \|\hat{r}\| 1 \|1\|^T \). 
\( \zeta > 0 \) and \( \beta > 0 \) are constant diagonal matrices, 
\( 0 < \gamma < 1 \) is constant, and \( \eta_i > 0 \) are constants. 
Substituting equations (24) to (30) into equation (23), we obtain

\[
\dot{V}_2 \leq -s^T \zeta \xi - \beta |s|^\gamma + \frac{1}{\gamma} \left( \sum_{i=1}^{4} |\hat{k}_i| + \frac{1}{\eta_i} \right) 
\]

where \( \zeta_2 = \min(\zeta_{\min}, 1/2(\eta_{k1}^{\min} + \eta_{k2}^{\min} + \eta_{k3}^{\min} + \eta_{k4}^{\min})) > 0 \)
and \( \mu_2 = \frac{1}{\gamma} \left( \sum_{i=1}^{4} \eta_{ki}^{\min} \right) > 0 \).

Multiplying (31) by \( e^{-\zeta t} \) yields

\[
\frac{d}{dt} (V_2 e^{\zeta t}) \leq \mu_2 e^{\zeta t} 
\]

Integrating (32) over \([0, t] \) leads to \( 0 \leq V_2 \leq V_2(0) - (1/\zeta_2) \mu_2 e^{-\zeta t} + (1/\zeta_2) \mu_2 \).
Therefore, this means that all the error signals are semi-globally uniformly ultimately bounded.

**Remark 1.** In the designed controller in equation (24), we use the finite time-based control term\(^{20} \) instead of the relay control term, i.e. sign function given as \( \beta \text{sign}(s) \), which gives chattering in the control action, to alleviate chattering and guarantee finite-time convergence of the filtered error.

**Remark 2.** If the estimation is precise, i.e. \( k = \hat{k}, \)
\( F_i = \hat{F}_i \), and \( \hat{d}_i = \hat{d}_i \), then \( \hat{k} = \hat{k}, \)
\( \hat{F}_i = \hat{F}_i \). Equation (31) can be rewritten as

\[
\dot{V}_2 \leq -s^T \zeta \xi - \beta |s|^\gamma + 2 \zeta \dot{V}_2 
\]

where \( \zeta' = \zeta_{\min}^{-1} \) and \( \gamma' = (\gamma + 1)/2 \). Therefore, from Slotine and Li,\(^{20} \) the settling time \( t_s \) can be given as

\[
t_s \leq \frac{1}{2 \zeta'(1 - \gamma')} \ln \frac{\zeta' V_2^{\frac{1}{\gamma'}}(0) + \beta}{\beta} 
\]

**Robust adaptive control**

The adaptive laws given in the previous section cannot estimate all the unknown parameters precisely
though robust control is inserted in equation (24). If the estimation errors are large, the control gain is high and larger control inputs appear in the actuator. Therefore, to provide robustness to the incomplete estimation and improve tracking control performance, the following disturbance compensator is included in the controller

\[ u_d(t) = \hat{\rho}s(F_{\rho}(t) - |s(t)|)^{-1} \]  

(35)

where \( \hat{\rho} \) and \( F_{\rho}(t) \) are defined later. This concept is borrowed from the funnel control16,17 as shown in Figure 3. If the boundary \( F_{\rho}(t) \) satisfies the funnel condition in equation (36) with \( |s(0)| < F_{\rho}(0) \)

\[ F_{\rho} : t \rightarrow \{ s \in R^d | F_{\rho}^{-1} \cdot |s| < 1 \} \]  

(36)

Figure 3. Basic concept of the Funnel control.

The scale factor \( \hat{\rho} \) of equation (35) is adjusted in order to ensure that the filtered error surface \( s(t) \) evolves inside the prescribed boundary function \( F_{\rho}(t) \). Thus, as the filtered error \( s(t) \) approaches the boundary \( F_{\rho}(t) \), the control action of \( u_d(t) \) increases, and as the error \( s(t) \) becomes small, the control action of \( u_d(t) \) decreases conversely. A proper boundary to constrain the uncertainty is selected by

\[ F_{\rho}(t) = (\epsilon_0 - \epsilon_{\infty}) \exp(-at) + \epsilon_{\infty} \]  

(37)

where \( \epsilon_0 = F_{\rho}(0) \geq \epsilon_{\infty} > 0 \), \( \epsilon_{\infty} = \liminf_{t \to \infty} F_{\rho}(t) \), and \( a > 0 \) is the decay rate of the boundary function. It is known that \( \epsilon_0 > |s(0)| \) regulates the transient time response, and \( \epsilon_{\infty} \) affects the steady-state time response.

Therefore, we adopt the following modified control law that guarantees robust control if the initial uncertainties are located within the boundaries of the prescribed constraint function. Considering the modeling errors and disturbance, equation (21) can be written as

\[ M(q)\ddot{s} = -C(q, \dot{q})s - M(q)\ddot{r} - C(q, \dot{q})\dot{r} \]

\[ -G(q) - F_{\rho}(t) + u_d(t) + \tilde{d}_p \rho + \tilde{d}_l(1-p) \]  

(38)

where the uncertainty is defined as \( F_{\rho}(t) = \Delta M(q)\ddot{r} + \Delta C(q, \dot{q})\dot{r} + \Delta F_{\rho}(q, \dot{q}) + F_{\rho}(t) \), \( \Delta(\cdot) \) are the expected adaptation perturbations of each dynamic term, \( |F_{\rho}| \leq \rho \), \( \rho \) is unknown upper bound of \( F_{\rho} \). Define the following Lyapunov function candidate as

\[ V_3 = \frac{1}{2} s^T M(q)s + \sum_{i=1}^{4} \frac{1}{2\eta_i} \tilde{d}_i^2 F_i + \frac{1}{2\eta_f} \tilde{F}_f + \frac{1}{\eta_p} \tilde{\rho}^T \tilde{\rho} \]  

(39)

where \( \tilde{\rho} = \rho - \hat{\rho} \), \( \hat{\rho} \) is estimate of \( \rho \). The time derivative of equation (39) is given similarly as follows

\[ \dot{V}_3 \leq s^T \left[ \tilde{K}^T \phi + \tilde{F}_f + u_d(t) - k_i \left( \frac{1}{\eta} s \right) \right] - \tilde{K}_k \left( \frac{1}{\eta} s \right) - \tilde{K}_k \left( \frac{1}{\eta} s \right) \]

(40)

The controller and adaptive law with consideration of the adaptive laws previously selected in equations (25) to (30) is selected as

\[ u_d = -\xi s - \tilde{K}^T \phi + \tilde{F}_f - \beta |s| \text{sgn}(s) - u_d(t) \]  

(41)

\[ \hat{\rho} = \Gamma_{\rho} \left( |s| \tilde{F}_f - |s| \right)^{-1} - s \tilde{\rho} \]  

(42)

Equation (40) is then written as

\[ \dot{V}_3 \leq -s^T \xi s - \beta |s|^{\gamma-1} - \sum_{i=1}^{4} \frac{\eta_i d_i^2}{2} + \sum_{j=1}^{r} \frac{\eta_f \tilde{F}_f^2}{2} + \sum_{k=1}^{l} \frac{\eta_p \tilde{\rho}^2}{2} \]

\[ + \rho \left( |s| - \rho \right) |s|^{\gamma-1} - \sum_{i=1}^{4} \frac{\eta_i d_i^2}{2} + \sum_{j=1}^{r} \frac{\eta_f \tilde{F}_f^2}{2} + \sum_{k=1}^{l} \frac{\eta_p \tilde{\rho}^2}{2} \]  

(43)
for a constant \( \alpha \), equation (43) can be rewritten as

\[
\eta_j^2 F_j^2 \leq -\lambda_{\min}(\alpha - 2\alpha) |s|^2 - \frac{4}{\sum_{j=1}^{r}} \eta_j^2 F_j^2
\]

where \( \lambda_{\min}(\alpha - 2\alpha) \) is bounded for \( s \to 0 \) and \( \alpha > 0 \) and

\[
\mu_3 = \frac{1}{2} \left( \sum_{k=1}^{r} \eta_k^2 \right)^{2} - \frac{\eta_j^2 \rho^2 \hat{\rho}}{2} + \rho F_\rho + \frac{1}{2} |\rho|^2
\]

Remark 3. In fact, the robustness to uncertainty obtained from the controllers and the adaptive laws in equations (35) and (42) is conditionally guaranteed provided that the prescribed uncertainty boundary function, \( F_\rho(t) \), satisfies \( |s| < F_\rho(0) \) or \( \epsilon_0 > |s(0)| \). Next, the uncertainty suppression of the control input of \( u_i(t) \) depends on the value of the scale factor \( \hat{\rho}(t) \). The proper estimation for uncertainty and selection of \( F_\rho(t) \) improve the robustness of the proposed controller.

Remark 4. If the condition of \( |s| < F_\rho(0) \) is violated, the proposed uncertainty compensation property disappears and the controller becomes the

![Figure 4. XY table system. (a) XY table and controller. (b) Schematic description of the ball-screw.](image)

<table>
<thead>
<tr>
<th>Table 1. XY table specification.</th>
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<tbody>
<tr>
<td>Component</td>
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<td>---------------------</td>
</tr>
<tr>
<td>Servo Motor</td>
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<tr>
<td>Servo Amplifier</td>
</tr>
<tr>
<td>Lead of ball-screw</td>
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<tr>
<td>Resolution of linear encoder</td>
</tr>
<tr>
<td>Motor power</td>
</tr>
</tbody>
</table>

The following inequalities are easily proved

\[
-\rho |s|^2(F_\rho - |s|)^{-1} \leq \rho(F_\rho + |s|) \quad (44)
\]

\[
|s| \rho \leq \kappa |s|^2 + \frac{1}{4\kappa} |\rho|^2 \quad (45)
\]

for a constant \( \kappa > 0 \). Using the relationships (44) and (45), equation (43) can be rewritten as

\[
\dot{V}_3 \leq - (\alpha - 2\alpha) |s|^2 - \beta|s|^{\alpha+1} - \sum_{j=1}^{r} \eta_j^2 F_j^2 - \sum_{j=1}^{r} \eta_j^2 F_j^2 F_j
\]
conventional terminal sliding mode controller. In this case, the trade-off gain tuning between chattering and robustness is considered to obtain a desirable performance. However, the controller term of $\beta|s|\text{sign}(s)$ in equation (42) does not induce larger chattering compared to the term of $\beta\text{sign}(s)$ adopted frequently in the conventional first-order SMC scheme.

**Remark 5.** The general selection rule for the constraint boundary function $F_c(t)$ does not exist, but this depends on the designer’s experience and trial

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**Figure 5.** Experimental results for the XY table system. (a) Estimated dynamic parameter in the X axis. (b) Estimated dynamics parameters in the Y axis. (c) Estimated friction parameters in the X axis. (d) Estimated friction parameters in the Y axis. (e) $\hat{\rho}_x$ and $\hat{\rho}_y$. (f) Control inputs of the robust adaptive control system in the XY axis. (g) Tracking errors of each control system in the X axis. (h) Tracking errors of each control system in the Y axis. (i) Tracking errors of the adaptive and robust adaptive control systems in the X axis. (j) Tracking errors of the adaptive and robust adaptive control systems in the Y axis.
Figure 5. Continued.
and error method according to variations of control system.

**Experimental evaluations**

In this section, the experimental applications for a XY table and an articulated manipulator are presented to verify the control performance of the proposed control strategy. Several controllers were designed: adaptive controller without deadzone and/or friction compensation and with deadzone and friction compensation, and robust adaptive controller with deadzone and friction compensation.

**Experiment for the XY table system**

The XY table system is shown in Figure 4, where an AC servo motor, an inverter and an universal motor driver produced by Mitsubishi company are equipped in the actuator system. The linear encoders attached in the side of linear motion guides of each axis measure the information regarding the position. The ball-screw transfer mechanism produced by Samick company is described in Figure 2. The complete specification of the XY table system is listed in Table 1. The MF 624 DSP control board combined with Matlab realtime toolbox software is used to transform input/output signals, and implement the design control algorithm.

The parameters of the controller and adaptive laws are selected as follows: \( \xi_x = 10, \quad \xi_y = 10, \quad \beta_x = 10, \quad \beta_y = 5, \quad \gamma_i = 0.5, \quad i = x, y, \quad \eta_{x1} = 1.5, \quad \eta_{x2} = 1, \quad \eta_{y1} = 1.2, \quad \eta_{y4} = 1.2, \quad \eta_{y1} = 1.5, \quad \eta_{y2} = 1, \quad \eta_{y3} = 1.5, \quad \eta_{y4} = 1.5, \quad \eta_{x1}' = 0.005, \quad \eta_{x2}' = 0.001, \quad \eta_{x3}' = 0.002. \)

<p>| Table 2. RMS tracking error of the adaptive and robust adaptive control system in the XY table. |
|-----------------|-----------------|-----------------|</p>
<table>
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<tr>
<th>Axis</th>
<th>Adaptive</th>
<th>Robust Adaptive</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.009 mm (100%)</td>
<td>0.007 mm (78%)</td>
</tr>
<tr>
<td>Y</td>
<td>0.027 mm (100%)</td>
<td>0.010 mm (37%)</td>
</tr>
</tbody>
</table>

\( \eta_{y4}' = 0.002, \quad \eta_{y1}' = 0.005, \quad \eta_{y2}' = 0.0025, \quad \eta_{y3}' = 0.0025, \) and \( \eta_{x4}' = 0.001. \) The performance functions were selected as follows:

\[
\begin{align*}
\varepsilon_x(t) &= (0.2 - 0.001)e^{-\xi_x t} + 0.001(mm), \\
\varepsilon_y(t) &= (0.2 - 0.002)e^{-\xi_y t} + 0.002(mm)
\end{align*}
\]

The command inputs are selected as \( q_{da}(t) = 2 \sin 0.2 t \) (mm) and \( q_{db}(t) = 2 \sin 0.25 t \) (mm). The estimated dynamic parameters and friction parameters are shown in Figure 5(a) to (d); Figure 5(e) presents the estimated uncertainty. The tracking errors without or with deadzone and friction compensation are presented in Figure 5(g) and (h), where the effect of deadzone compensation is greater than friction compensation effect. The tracking errors for robust control are shown in Figure 5(i) and (j), where the root mean square (RMS) error of the proposed control decreased to maximum 37% of that of the adaptive control system, as shown in Table 2. Therefore, the proposed uncertainty compensator suppresses the uncertainty effectively. Figure 6(f) shows the control inputs of the robust adaptive control system in each axis.

**Experiment for the articulated manipulator system**

As the second application, the experiment for the articulated robot manipulator presented in Figure 6 was carried out to prove the efficacy of the proposed control strategy. In this application, two controllers were designed: the adaptive controller and robust adaptive controller. The designed controllers were implemented through the Matlab realtime toolbox with MF624 control board equipped with D/A and A/D converter. The angles of each link are measured by the rotary encoder attached to each DC servo motor.

The parameters of the controller and adaptive laws are selected as follows: \( \xi_2 = 120, \quad \xi_2 = 90, \quad \beta_1 = 60, \quad \beta_2 = 60, \quad \gamma_i = 0.5, \quad i = 1, 2, \quad \eta_{11} = 5, \quad \eta_{12} = 2.5, \)

Figure 6. Photograph and diagram of the articulated manipulator control system.
The performance functions were selected as follows

\[
\begin{align*}
\eta_1(t) &= (0.1 - 0.005)e^{-t} + 0.005 \text{ (rad)} \\
\eta_2(t) &= (0.1 - 0.0025)e^{-t} + 0.0025 \text{ (rad)}
\end{align*}
\]  

The command inputs are selected as

\[q_d(t) = 0.1 \sin 0.4t \text{ (rad)} \quad \text{and} \quad q_d(0) = 0.15 \sin 0.4t \text{ (rad)}\]

In Figure 7(a) to (g), the estimated results for the dynamic parameters, friction, deadzone, and uncertainty are shown. The tracking outputs of the proposed control are presented in Figure 7(h) and (i) and the tracking errors of the adaptive and robust adaptive control systems are shown in Figure 7(j) and (k), and listed in Table 3. The error size in the proposed control system decreased to 38% of that of the adaptive control system. Therefore, it is proved that the proposed control scheme has more advanced control performance than the adaptive control system.

**Conclusions**

In this paper, the adaptive control scheme with funnel robust compensation was developed to provide a high enhanced position tracking performance.
Figure 7. Continued.
of the nonlinear servo dynamic system in the presence of both deadzone and friction. First, an adaptive controller was designed using the adaptive laws for estimation of the dynamic parameters of the servo system, Tustin friction, and deadzone parameters. Next, a funnel compensator was added to enforce the robustness of the adaptive controller. From the Lyapunov stability theorem, adaptive laws for the controller, friction, and deadzone observes were derived. As design examples, the XY table and the robotic manipulator in the presence of friction and deadzone were chosen. The favorable position tracking performance of the proposed control scheme was validated from experiments by its effective compensation for the deadzone, friction, and uncertainties.

Declaration of Conflicting Interests
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References


