Reference Slip Ratio Generation and Adaptive Sliding Mode Control for Railway Rolling Stocks

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We propose a reference slip ratio generation algorithm that accounts for a large adhesion force to improve the braking performance of railway rolling stocks even if the rail conditions change. Our algorithm is based on fuzzy logic, the efficiency of which was evaluated by comparing the braking distances of rolling stocks using the proposed algorithm and using constant reference slip ratios under various rail conditions. Our proposed slip ratio generation algorithm was used as the basis of an adaptive sliding mode controller for a rolling stocks quarter model. In this design, an adaptive rule was developed using the Lyapunov stability theorem, and the performance of the proposed control system was evaluated by computer simulation.

1. Introduction

Economic and technological developments have increased the importance of rapid transportation systems, including railways. The capacity and speed of railway rolling stocks has been improved through advances in electric railway propulsion systems, including the insulated gate bipolar transistor and the variable-voltage variable-frequency inverter. This has put increased emphasis on the comfort and safety of passenger railway systems. Railway brake systems, in particular, are important in this regard because the brake system of railway rolling stocks is directly related to passenger safety.

Railway rolling stocks brake systems fall into two groups: electrical brakes that are used for higher speed systems, and mechanical brakes that are used for lower speed systems. In actual electrical braking mechanisms, however, the braking force is a blend of both the electrical and mechanical brake systems, which are applied in a manner appropriate to the braking conditions.

In their investigations of rolling stocks braking characteristics, many researchers have studied the adhesion force, which is the friction force between the rail and the wheel. The adhesion force has nonlinear characteristics, which have been demonstrated in many experiments. The adhesion coefficient increases to a maximum value in the creep region and decreases in the slip region depending on the slip ratio. In addition, the value of the adhesion coefficient on dry rail is generally larger than that on wet rail at the same slip ratio.

It is very difficult to measure the adhesion coefficient in real time, which would be useful for the actual design of brake systems. Observer techniques have been used to estimate the adhesion force, wheel-slip brake control systems are designed based on this estimate. Anti-lock brake control systems have also been designed based on the estimated adhesion force. However, these control systems cannot reflect changes in the adhesion coefficient according to the rail condition. For this reason, Kawamura et al. proposed a reference slip ratio generation algorithm that can reflect various rail conditions by using a variable reference slip ratio generator. In this technique, the generated reference slip ratio was mainly affected by the noise signal from the wheel angular velocity sensor, and the system uncertainties of the rolling stocks, such as the driving resistance and mass, were not considered. Therefore, this algorithm cannot guarantee robust performance and stability.
Fuzzy logic algorithms are another possibility for rolling stocks brake systems.\(^1\)

The brake performance in previous research is directly related to the adhesion force, which is proportional to the adhesion coefficient. Improved brake performance requires a large adhesion coefficient. Therefore, in this paper, we propose a reference slip ratio generator that uses fuzzy logic\(^{12,13}\) to obtain the maximum adhesion coefficient. In addition, an adaptive sliding mode controller is designed based on the Lyapunov stability theorem that takes into account rolling stocks system uncertainties. The performance of the adaptive sliding mode control system with the proposed slip ratio generator was evaluated by computer simulation.

The rest of this paper is organized as follows. Section 2 describes the behavior of the adhesion coefficient as a function of the slip ratio and presents the proposed reference slip ratio generation algorithm. Section 3 describes the design of an anti-slip brake control system using the adaptive sliding mode control technique. Section 4 describes the computer simulations for the proposed control system with the proposed slip ratio generator. Section 5 presents the concluding remarks.

2. Reference slip ratio generation for a brake system

Railway rolling stocks dynamics can be simplified into a quarter model by assuming that the rolling stocks travels in the longitudinal direction without lateral motion. Figure 1 shows a schematic diagram of the rolling stocks quarter model, and Eqs. (1) and (2) give the related equations of motion. Equations (3) and (4) give the adhesion force and the slip ratio, respectively.

\[
\begin{align*}
J\ddot{\omega} &= -Bo + rF_a - T_b \\
Mr &= -F_a \\
F_a &= \mu(\lambda)N \\
\dot{\lambda} &= \frac{v - r\omega}{v}
\end{align*}
\]

Here, \(J\) is the inertia of the wheel; \(M\) is the rolling stocks mass; \(B\) is the viscous friction torque coefficient between the brake pad and the wheel; \(F_a\) is the adhesion force; \(T_b\) is the brake torque; \(\mu(\lambda)\) is the coefficient of the adhesion force; \(N\) is the normal force at the wheel surface contact; \(\lambda\) is the slip ratio; \(v\) and \(r\omega\) are the velocities of rolling stocks and the wheel, respectively; and \(r\) is the radius of the wheel.

From Eqs. (1)–(4), it is clear that the brake performance of the rolling stocks depends on the adhesion force, which is proportional to the adhesion coefficient \(\mu(\lambda)\), which depends on the rolling stocks slip ratio.

Figure 2 shows a typical relationship between the adhesion coefficient and the slip ratio according to the rail condition. The adhesion coefficient increases in the creep region and decreases in the slip region as a function of the slip ratio. In addition, the value of the adhesion coefficient on dry rail is larger than that on wet rail.

In real braking mechanisms, the braking occurs in the creep region and slipping occurs in the slip region. A brake force that is larger than the adhesion force tends to flatten wheels in the slip region. Therefore, slip control is required in a brake control system to prevent wheel flatness. However, because it is very difficult to measure the adhesion force in real time, the constant reference slip ratio is used in most actual brake control systems.

The relationship between the adhesion force and the slip ratio should be considered in the brake control system to improve the brake performance and prevent wheel flatness. We propose a reference slip ratio generation algorithm is proposed to maintain a large adhesion force. Fuzzy logic based on linguistic information are used as the conditions for obtaining the maximum adhesion force, and the updating rule of the reference slip ratio is selected to be

\[
\dot{\lambda}^{k+1} = \dot{\lambda}^k + \Delta(\dot{\lambda}, \dot{\mu}),
\]

where \(\Delta(\dot{\lambda}, \dot{\mu})\) is calculated by fuzzy logic. The function \(\Delta(\dot{\lambda}, \dot{\mu})\) represents the variation of the derivative of the adhesion coefficient with respect to the slip ratio.

As Fig. 2 shows, the maximum adhesion coefficient occurs when \(d\mu/d\dot{\lambda}\) is zero. Therefore, \(\Delta(\dot{\lambda}, \dot{\mu})\) can be referred to the value of \(d\mu/d\dot{\lambda}\), and the membership function can be selected based on it. The input linguistic values of \(d\mu/d\dot{\lambda}\) and the output linguistic values are divided into four regions. Figure 3 shows these membership functions. In the figure, the linguistic values of NB, N, P, and PB indicate negative-big, negative, positive, and positive-big, respectively. The number of the membership function and the region are selected based on experimental results, which are related
to the adhesion coefficient. The input and output ranges are defined using the information of the adhesion coefficient for the wet and dry rail conditions. The scale of the output range is defined from −1 to 1. Table 1 shows the fuzzy logic defined on the basis of these input and output membership functions.

3. Anti-slip control for rolling stocks brake systems

The brake control system of rolling stocks should perform well in reducing the braking distance and preventing wheel flatness. This requires an anti-slip control system. This system must be robust because of the complex rolling stocks dynamics and the large uncertainty of the adhesion force.

![Input membership function](image1)

![Output membership function](image2)

Fig. 3 Input and output membership functions

Therefore, we use an adaptive sliding mode control scheme. The adaptive law is used because of the rolling stocks mass, which depends on the number of passengers on board and the friction coefficient between the wheel and the brake pad.

Figure 4 shows a block diagram of the anti-slip brake control system with the fuzzy reference slip ratio generator. This uses the reference slip ratio obtained by Eq. (5), which varies according to the rail condition. The sliding surface $s(t)$ for the sliding mode controller is defined as

$$s(t) = e + \gamma \int_{0}^{t} edt,$$  

where $e = \lambda_{ref} - \lambda$ and $\gamma$ is a positive definite design parameter.

Then, the derivative of the sliding surface can be expressed as

$$\dot{s} = \dot{\lambda}_{ref} - \frac{(v - \dot{\omega})v - (\dot{v} - r\dot{\omega})\dot{v}}{v^2} + \gamma e.$$  

Based on rolling stocks dynamics, the derivative of the sliding surface can be rewritten as

$$\dot{s} = \dot{\lambda}_{ref} + \frac{r^2}{Jv} F_{a} - \frac{r}{Jv} T_{b} - \frac{rb}{Jv} \omega - \frac{1}{Mv} (\lambda - 1) F_{a} + \gamma e.$$  

Since the adhesion force cannot be measured in real time, the adhesion force observer should be designed to estimate the adhesion force as

Table 1 Fuzzy logic for the generation of $\Delta$

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>P</td>
<td>P</td>
</tr>
<tr>
<td>PB</td>
<td>PB</td>
</tr>
</tbody>
</table>

![Block diagram](image3)

Fig. 4 Block diagram of the anti-slip brake control system with the fuzzy reference slip ratio generator

$$\dot{F}_{a} = \frac{1}{r} \left( \frac{Js}{\tau s + 1} - \omega T_{b} + B \omega \right)$$  

or

$$\dot{F}_{a} = \frac{1}{r} T_{b} \frac{J}{\tau e} \left( \frac{1}{J} + \frac{rB}{J} - \frac{1}{\tau s + 1} \right) \omega$$  

where $\tau$ is the time constant of the first-order filter in the adhesion force observer.

To select the control law that can satisfy the reaching condition and solve the chattering problem of the control input, $\dot{s}$ can be represented as

$$\dot{s} = \dot{\lambda}_{ref} + \frac{r^2}{Jv} \dot{F}_{a} - \frac{r}{Jv} \dot{T}_{b} - \frac{rb}{Jv} \dot{\omega} - \frac{1}{Mv} (\lambda - 1) F_{a} + \gamma e$$  

$$\dot{s} = -D_{s} - K \text{sgn}(s)$$

where $D$ and $K$ are positive definite design parameters. Also, Eq. (11) can be rewritten as

$$\dot{s} = \dot{\lambda}_{ref} - \frac{r}{Jv} \dot{T}_{b} + \left( \frac{r^2}{J} + \frac{1}{M} \right) \frac{\lambda_{a}}{v} \frac{rb}{Jv} \omega - \frac{1}{Mv} F_{a} + \gamma e$$  

$$\dot{s} = \dot{\lambda}_{ref} - \frac{r}{Jv} \dot{T}_{b} + \gamma e + \theta^{T} \varphi = -D_{s} - K \text{sgn}(s)$$
where \( \varrho^T = \begin{bmatrix} \frac{r^2}{J} + \frac{1}{M} & -\frac{r B}{J} & -\frac{1}{M} \end{bmatrix} \) and \( \varphi^T = \begin{bmatrix} \frac{F_u}{v} \omega \frac{\lambda F_u}{v} \end{bmatrix} \).

The parameter vector \( \vartheta \) is unknown, and is estimated using the update law. From Eq. (12) and the estimated unknown parameter vector \( \hat{\vartheta} \), the estimated sliding mode control law can be selected as follows:

\[
\hat{T}_b = \frac{J_t}{r} \left( \hat{\vartheta} \right)_{\text{ref}} + \gamma e + \hat{\vartheta}^T \varphi + D_s + K_s \text{sgn}(s))
\]  

To obtain the update law for the unknown parameters, the Lyapunov candidate function is defined as \( V = \frac{1}{2} \hat{\vartheta}^T \hat{\vartheta} + \frac{1}{2 \sigma} \hat{\varphi}^T \hat{\varphi} \)

where \( \hat{\vartheta} = \vartheta - \hat{\vartheta} \); \( \hat{\vartheta} \) and \( \hat{\varphi} \) are the nominal and estimated parameter vectors, respectively; and \( \sigma \) is a positive definite design parameter. The derivative of the Lyapunov function, including sliding dynamics, is

\[
\dot{V} = s \hat{\vartheta}^T + \frac{1}{\sigma} \hat{\varphi}^T \hat{\varphi}
\]

or

\[
\dot{V} = s \left( \hat{\vartheta}_{\text{ref}} - \frac{r}{J_t} \hat{T}_b + \gamma e + \hat{\vartheta}^T \varphi \right) - \frac{1}{\sigma} \hat{\varphi}^T \hat{\varphi}
\]

Substituting the control input \( \hat{T}_b \) of Eq. (13) into Eq. (16) gives

\[
\dot{V} = -D_s \hat{\vartheta}^2 - K_s \text{ssgn}(s) + \frac{1}{\sigma} \hat{\varphi}^T \hat{\varphi}.
\]

From Eq. (17), the update law for the unknown parameters is selected to be

\[
\dot{\hat{\vartheta}} = \sigma \hat{\vartheta} \hat{\varphi}
\]

Then, the derivative of the Lyapunov function is given by Eq. (19) and the asymptotic stability of the adaptive sliding mode brake control system is guaranteed.

\[
\dot{V} = -D_s \hat{\vartheta}^2 - K_s \text{ssgn}(s) \leq 0
\]

### 4. Simulation results

The performance of the adaptive sliding mode brake control system was evaluated using a Matlab computer simulation using the parameters listed in Table 2. For the purposes of the simulation, we assumed that the rolling stocks moves through periodic wet and dry conditions. The relationship between the adhesion coefficient \( \mu(\lambda) \) and the slip ratio can be expressed as

\[
\mu(\lambda) = \frac{a}{1 - e^{-b\lambda}} - \frac{\lambda}{10},
\]

where \( a \) and \( b \) are parameters that change according to the rail condition. Parameters \( a \) and \( b \) were set to 0.275 and 40 for the wet rail, 0.375 and 15 for the dry rail, and 0.32 and 25 for the intermediate condition, respectively. These are plotted in Fig. 5.

We assumed that the initial braking velocity of the rolling stocks was 100 km/h and the rolling stocks encountered a different rail condition every 5 s. Figure 6 shows the generated reference slip ratio based on these assumptions. Large adhesion coefficient conditions could be maintained for any traveling condition using the generated reference slip ratio. The maximum adhesion coefficient was obtained when the slip ratio was approximately 0.14, 0.25, and 0.17 for dry, intermediate, and wet rail conditions, respectively.

Figure 7 shows the locus of the adhesion coefficient as a function of the reference slip ratio. Based on these reference slip ratios, the proposed control scheme was applied to the rolling stocks quarter model and computer simulations were conducted to evaluate the performance of the proposed slip control system with the fuzzy reference slip ratio generator. Figure 8 shows the velocities of rolling stocks and the wheel under these conditions.

Figure 9 compares the rolling stocks velocities using the proposed and constant reference slip ratios to evaluate the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertial moment of the wheel</td>
<td>J</td>
<td>240 kg•m²</td>
</tr>
<tr>
<td>Radius of the wheel</td>
<td>r</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Mass of the car body</td>
<td>M</td>
<td>9870 kg</td>
</tr>
</tbody>
</table>

Fig. 5 Adhesion coefficient as a function of the slip ratio and rail condition

Fig. 6 Generated reference slip ratio for an changeable rail condition
5. Conclusions

Railway rolling stocks mechanical brake systems are influenced by the adhesion force between the rail and the wheel. A reference slip ratio generation algorithm using fuzzy logic was proposed to account for this influence. A brake control system was designed using the adaptive sliding mode control scheme based on the proposed reference slip ratio, and an adaptive law selected for the unknown parameters was developed using the Lyapunov stability theorem. The proposed reference slip ratio algorithm was applied to the rolling stocks’ quarter model for various rail conditions. The required large adhesion coefficient was maintained for all traveling conditions through the locus of the generated reference slip ratio. In addition, the rolling stocks velocities for the constant and proposed reference slip ratios were compared for various rail conditions to evaluate the performance of the proposed control scheme. The simulation results indicated that the proposed adaptive sliding mode anti-slip brake control system with the fuzzy reference slip ratio generator quickly reduced the velocity of the rolling stocks and had a shorter braking distance than conventional systems.

REFERENCES
6. Ohishi, K., Nakano, K., Miyashita, I. and Yasukawa, S., “Anti-


