Robust position control of electro-hydraulic actuator systems using the adaptive back-stepping control scheme

H M Kim¹, S H Park¹, J H Song¹, and J S Kim²*

¹School of Mechanical Engineering, Pusan National University, Busan, Republic of Korea
²School of Mechanical Engineering, Pusan National University, Busan, Republic of Korea

The manuscript was received on 18 December 2009 and was accepted after revision for publication on 9 April 2010.

DOI: 10.1243/09596518JSCE980

Abstract: In general, the position control of electro-hydraulic actuator (EHA) systems is difficult because of system uncertainties such as Coulomb friction, viscous friction, and pump leakage coefficient. Even if the exact values of the friction and pump leakage coefficient may be obtained through experiment, the identification procedure is very complicated and requires much effort. In addition, the identified values may not guarantee the reliability of systems because of the variation of the operating condition. Therefore, in this paper, an adaptive back-stepping control (ABSC) scheme is proposed to overcome the problem of system uncertainties effectively and to improve the tracking performance of EHA systems. In order to implement the proposed control scheme, the system uncertainties in EHA systems are considered as only one term. In addition, in order to obtain the virtual controls for stabilizing the closed-loop system, the update rule for the system uncertainty term is induced by the Lyapunov control function (LCF). To verify the performance and robustness of the proposed control system, computer simulation of the proposed control system is executed first and the proposed control scheme is implemented for an EHA system by experiment. From the computer simulation and experimental results, it was found that the ABSC system produces the desired tracking performance and has robustness to the system uncertainties of EHA systems.

Keywords: electro-hydraulic actuator, LuGre friction model, adaptive back-stepping control

1 INTRODUCTION

Conventional hydraulic actuator (CHA) systems have been widely used as power units because they can generate very large power compared with their size. In general, a CHA system consists of an electric motor, a pump, a reservoir, various valves, hoses, which are used to transfer the working fluid, and an actuator. CHA systems, however, have some problems such as environmental pollution caused by the leakage of the working fluid, maintenance load, heavy weight, and limited installation space. These shortcomings can be overcome by compactly integrating the components of CHA systems and by applying a suitable control scheme for the electric motor. To overcome these shortcomings of CHA systems, electro-hydraulic actuator (EHA) systems have been developed, having merits such as smaller size, higher energy efficiency, and faster response than existing CHA systems. However, for the robust position control of EHA systems, system uncertainties such as the friction between the piston and cylinder and the pump leakage coefficient have to be considered.

To solve these system uncertainty problems of EHA systems and to achieve the robustness of EHA systems with system disturbance and bounded parameter uncertainties, Wang et al. presented a sliding mode control and a variable structure filter based on the variable structure system. Perron et al. proposed a sliding mode control scheme for the
robust position control of EHA systems showing volumetric capacity perturbation of the pump [3]. However, these control methods have some chattering problem owing to the variable structure control scheme. The chattering vibrates the system and may reduce the life cycle of the system. Jun et al. presented a fuzzy logic self-tuning PID controller for regulating the BLDC motor of EHA systems which has non-linear characteristics such as the saturation of the motor power and dead-zone owing to the static friction [4]. Chinniah et al. used a robust extended Kalman filter, which can estimate the viscous friction and effective bulk modulus, to detect faults in EHA systems [5]. Kaddissi et al. applied a robust indirect adaptive back-stepping control (ABSC) scheme to EHA systems having perturbations of the viscous friction coefficient and the effective bulk modulus owing to temperature variations [6]. However, in spite of the variation of the effective bulk modulus due to the temperature and pressure variations, Chinniah et al. considered only the case of constant effective bulk modulus and Kaddissi et al. used EHA systems that are not controlled by an electric motor but by a servo valve.

In this paper, an ABSC scheme was proposed for EHA systems to obtain the desired tracking performance and the robustness to system uncertainties. First, to realize a stable back-stepping control (BSC) system with a closed-loop structure and to select new state variables, EHA system dynamics are represented with state equations and error equations. Defining the Lyapunov control functions, a BSC system can be designed, which can guarantee exponential stability for the nominal system without system uncertainties. However, the BSC system cannot achieve robustness to system uncertainties. To overcome the drawback of the BSC system, an ABSC scheme for EHA position control systems with classical discrete disturbance observer was proposed. To evaluate the tracking performance and robustness of the proposed EHA position control system, both BSC and ABSC schemes were evaluated by computer simulation and experiment.

2 SYSTEM MODELLING OF EHA SYSTEMS

Figure 1 shows the simplified schematic diagram of an EHA system that consists of an electric servo motor, bi-directional gear pump, and actuator. The servo motor rotates the gear pump, which, in turn, generates the flowrate. The pressure generated by the flowrate changes the position of the piston rod. The movement direction of the piston is related to the rotational direction of the servo motor. The chamber volumes of the actuator depend on the cross-sectional area and the displacement of the piston as follows

\[
\begin{align*}
V_A(t) &= V_{0A} + Ax(t) \\
V_B(t) &= V_{0B} - Ax(t)
\end{align*}
\]

where \( V \) and \( V_0 \) are the chamber volume and the initial chamber volume respectively, \( A \) and \( x \) are the pressure area of a double rod hydraulic cylinder and displacement of the piston respectively, and subscripts A and B denote the chamber notations of the actuator.

Considering the fluid compressibility and continuity principle for the actuator, the flowrate equations of both ports of the actuator can be represented as [7]

\[
\begin{align*}
Q_A &= Ax + \frac{V_{0A} + Ax}{\beta_e} P_A + LP_A \\
Q_B &= Ax - \frac{V_{0B} - Ax}{\beta_e} P_B - LP_B
\end{align*}
\]

where \( Q \) is the flowrate in the actuator, \( \beta_e \) is the effective bulk modulus of the working fluid, and \( L \) and \( P \) are the actuator external leakage coefficient and the pressure in the chamber respectively.

It is assumed that there is no fluid leakage of conduits because the conduits of EHA systems are very short and hard. Then, equation (2) can be expressed as

\[
\begin{align*}
Q_A &= Ax + \frac{V_{0A} + Ax}{\beta_e} P_A \\
Q_B &= Ax - \frac{V_{0B} - Ax}{\beta_e} P_B
\end{align*}
\]

The electric motor, directly connected to the hydraulic pump, changes the flow direction and...
adjusts the flowrate through the ports. In addition, the pressure generated by the continuous supply of flow in the actuator can produce a minute fluid leakage of the pump. Hence, the equations for the fluid leakage of the pump are expressed as

\[
\begin{align*}
Q_a &= C_p \omega_p - L_d P_l \\
Q_b &= -Q_a
\end{align*}
\] (4)

where \(Q\) is the flowrate of the pump, whose subscripts a and b denote the ports of the pump, \(C_p\) is the volumetric capacity of the pump, \(\omega_p\) is the rotational velocity of the electric motor, \(L_d\) is the leakage factor of the pump, and the load pressure \(P_l = P_A - P_B\). From equation (4), the inflow and outflow of the pump are expressed as functions of the rotational velocity \(\omega_p\).

In addition, the actuator dynamic equation of EHA systems is expressed as

\[
(P_A - P_B)A = M \ddot{x} + F_l + F_{ex}
\] (5)

where \(M\) and \(x\) are the mass and displacement of the piston respectively, \(F_l\) is the friction force between the cylinder and piston, and \(F_{ex}\) is the external disturbance force.

In order to substitute equation (3) into equation (5), the derivative of equation (5) is expressed as

\[
(\dot{P}_A - \dot{P}_B)A = M \ddot{x} + \dot{F}_l + \dot{F}_{ex}
\] (6)

In addition, it is assumed that the conduits connected between the actuator ports and the pump ports are very short. Then, the flowrates in equations (3) and (4) can be represented as \(Q_A = Q_a\) and \(Q_B = Q_b\). Substituting equations (1) to (4) into equation (6), therefore, the dynamic equation of EHA systems can be represented as

\[
\ddot{x} = -\frac{1}{M} \left\{ \beta_c A^2 \left[ \frac{1}{V_A} + \frac{1}{V_B} \right] \ddot{x} + \dot{F}_l + \dot{F}_{ex} \right\} \\
+ \frac{\beta_c A}{M} \left[ \frac{1}{V_A} + \frac{1}{V_B} \right] (L_d P_l - C_p \omega_p)
\] (7)

To represent the characteristics of the friction \(F_l\) between the piston and cylinder, the LuGre friction model is considered. The LuGre friction model is based on bristles analysis, which is represented with the average deflection force of bristles stiffness. The deflection displacement equation of bristles \(z\), which is actually unmeasurable by experiment, is expressed as \([8, 9]\)

\[
\frac{dz}{dt} = \frac{\sigma_0 |\dot{x}|}{g(x)} z
\] (8)

where \(\sigma_0\) is the bristles stiffness coefficient, \(z\) is the unmeasurable internal state, and the non-linear function \(g(x)\) depends on the material property, grade of lubrication, and temperature; that is

\[
g(x) = F_c + F_s e^{-\left(\frac{x}{\nu_s}\right)^2}
\] (9)

where \(F_c\), \(F_s\), and \(\nu_s\) represent the Coulomb friction force, static friction force, and Stribeck velocity between the cylinder and piston respectively.

If the relative velocity of the contact materials increases gradually, the friction force decreases instantaneously and then it increases gradually again; this effect is called the Stribeck effect and the relative velocity is the Stribeck velocity. This phenomenon depends on the material property, grade of lubrication and temperature. The friction force \(F_l\) can be represented with the average deflection \(z\) and the velocity of the piston \(\dot{x}\) as follows

\[
F_l = \sigma_0 z + \sigma_1 \dot{z} + \mu \dot{x}
\] (10)

where \(\sigma_1\) and \(\mu\) represent the bristles damping and viscous friction coefficients respectively.

### 3 CONTROLLER DESIGN FOR THE EHA POSITION CONTROL SYSTEMS

The EHA position control system consists of the inner loop for the angular velocity control of the servo motor/pump and the outer loop for the position control of the piston. For the velocity control of the motor in the inner loop, Kokotovic et al. applied an adaptive control scheme so that the EHA position control systems can have robustness [1]. Habibi and Goldenberg stated that if the inner loop dynamics are stable, the control gains of the proportional–integral–derivative controller (PID) velocity controller in the inner loop can have relatively large values and then the disturbance effect can be sufficiently rejected [10]. The velocity controller in the inner loop is very important because it regulates the electric motor. However, the case of reference [1] is very complicated and the case of reference [10], although it is theoretically possible, has a physical limitation that increases the control gains of the inner loop controller. Therefore, it is desirable to handle the controller in the outer loop rather than in
the inner loop to improve the performance and robustness of EHA position control systems.

In this paper, the BSC and ABSC schemes based on EHA system dynamics are considered as the position controller. First, to design a BSC system, equation (7) is transformed to a general form \[ a_{11} \] as follows

\[ x = f + bu \]  

where

\[ f = - \frac{1}{M} \left( \beta_e A^2 \left( \frac{1}{V} + \frac{1}{V^2} \right) x + \dot{F}_t + F_{ex} \right) - \beta_e A \left( \frac{1}{V} + \frac{1}{V^2} \right) p_L, \]

\[ b = - \frac{\beta_e A (V_A + V_B) C_p}{M V_A V_B}, \]

\[ u = \omega_p. \]

Now, let equation (11) represent state equations as follows

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= f + bu
\end{align*}
\]

And, in order to design the BSC system, new state variables are defined as follows

\[ z_1 = x_1 - x_d \]  

\[ z_2 = x_2 - x_1(z_1) \]  

\[ z_3 = x_3 - x_2(z_1, z_2) \]

where \( x_d \) is the desired position input, and \( z_1 \) and \( z_2 \) are the functions for new state variables, which can be obtained through the following BSC design procedure.

**Step 1**

From equation (13), the state equation for \( z_1 \) can be described as

\[ \dot{z}_1 = z_2 + x_1(z_1) - \dot{x}_d \]  

\[ x_1(z_1) \] is the virtual control which should be selected to guarantee the stability of the control system through the Lyapunov control function (LCF) which is defined as

\[ V_1(z_1) = \frac{1}{2} z_1^2 \]

Then

\[ \dot{V}_1(z_1) = z_1 \dot{z}_1 = z_1 [x_1(z_1) - \dot{x}_d] + z_1 \dot{z}_2 \]

From equation (18), if \( x_1(z_1) = -k_1 z_1 + \dot{x}_d \), equation (16) can be exponentially stable when \( t \rightarrow \infty \). And \( k_1 (> 0) \) is a design parameter.

**Step 2**

From equation (14), the state equation for \( z_2 \) can be described as

\[ \dot{z}_2 = z_3 + x_2(z_1, z_2) - \dot{x}_1(z_1) \]

where

\[ \dot{x}_1(z_1) = \frac{\partial^2 z_1}{\partial z_1^2} \dot{z}_1 + \frac{\partial^2 z_1}{\partial x_d} \dot{x}_d = -k_1 z_1 + \dot{x}_d \]

\[ = -k_1(z_2 - k_1 z_1) + \dot{x}_d \]

Since equation (19) includes the information of equation (16), the second LCF for obtaining the virtual control to guarantee the stability of the control system can be selected as

\[ V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2. \]

Then

\[ \dot{V}_2(z_1, z_2) = \dot{V}_1(z_1) + \dot{z}_2 \dot{z}_2 = -k_1 z_1^2 + z_2 \dot{z}_2 + z_2 [x_1 + x_2(z_1, z_2) - \dot{x}_1(z_1)] \]

(21)

If the virtual control \( x_2 \) in the last term of equation (21) is defined as

\[ x_2(z_1, z_2) = -k_2 z_2 - z_1 + \dot{x}_1(z_1) \]

where \( k_2 (> 0) \) is a design parameter, then another expression of \( x_2 \) can be rearranged as

\[ x_2(z_1, z_2) = -(k_1 + k_2) z_2 - (1 - k_1^2) z_1 + \dot{x}_d \]  

(22)
Therefore
\[ \dot{V}_2 = z_2 z_3 - k_1 z_1^2 - k_2 z_2^2 \]
(23)

\textbf{Step 3}

From equation (15), the state equation for \( z_3 \) is described as
\[ \dot{z}_3 = \dot{x}_3 - \dot{a}_2(z_1, z_2) = f + bu - \dot{a}_2(z_1, z_2) \]
(24)

where
\[ \dot{a}_2(z_1, z_2) = \frac{\partial a_2}{\partial z_1} z_1 + \frac{\partial a_2}{\partial z_2} z_2 + \frac{\partial a_2}{\partial x_d} x_d = -z_1 - k_2 z_2 + \dot{x}_{2} \]
(25)

and
\[ \dot{x}_1 = \frac{\partial x_1}{\partial z_1} z_1 + \frac{\partial x_1}{\partial z_2} z_2 + \frac{\partial x_1}{\partial x_d} x_d = k_1^2 z_1 - k_1 z_2 + \dot{x}_{d} \]
(26)

Substituting equations (16), (19), and (26) into equation (25), equation (25) can be rearranged as
\[ \dot{a}_2(z_1, z_2) = (k_1^2 - 1) z_1 - (k_1 + k_2) z_2 + \dot{x}_{d} \]
(27)

Since equation (24) uses the information of \( z_1 \) and \( z_2 \) owing to the property of the design procedure of the back-stepping control, the third LCF for equation (15) can be defined as
\[ V_3(z_1, z_2, z_3) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 \]
(28)

Then
\[ \dot{V}_3(z_1, z_2, z_3) = \dot{V}_2 + z_3 \dot{z}_3 \]
\[ = -k_1 z_1^2 - k_2 z_2^2 + z_3(f + bu - \dot{a}_2) \]
(29)

If the last term of equation (29) for satisfying the system stability is defined as
\[ -k_3 z_3 = z_3(f + bu - \dot{a}_2) \]
(30)

then the BSC law can be selected as
\[ u = \frac{1}{b}(\dot{a}_2 - k_3 z_3 - z_2 - f) \]
(31)

From equation (31), if the information of \( f \) is assumed to be known, the negative semi-definite of \( \dot{V}_3 \) can be obtained by substituting equation (31) into equation (29) as
\[ \dot{V}_3(z_1, z_2, z_3) = \dot{V}_2 + z_3 \dot{z}_3 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 \leq 0 \]
(32)

From equation (32), it is found that EHA position control systems using the BSC law of equation (31) can guarantee exponential stability.

If system uncertainties can be exactly known, the BSC law of equation (31) can achieve the desired tracking performance and the robustness to the system uncertainties of EHA systems. However, the BSC law of equation (31) will cause a tracking error and does not achieve the robustness to the system uncertainties because the value of \( f \) cannot be exactly known. To improve the tracking performance and the robustness to the system uncertainties, the value of \( f \), in which system uncertainties are included, should be estimated.

Therefore, in this paper, an ABSC scheme is proposed, which is the BSC scheme with the estimator of \( f \). In order to design the ABSC system, the BSC law of equation (31) is modified as
\[ u = \frac{1}{b}(\dot{a}_2 - k_3 z_3 - z_2 - \dot{f}) \]
(33)

where \( \dot{f} \) is the estimator of the system uncertainties.

Substituting equation (33) into equation (12), equation (12) is modified as
\[ \begin{aligned} 
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= \dot{f} + \dot{a}_2 - k_3 z_3 - z_2 
\end{aligned} \]
(34)

where \( \dot{f} = f - \dot{f} \).

From equations (13), (14), and (15), these equations are the error equations for the velocity, acceleration and jerk of the piston, which include \( x_1(z_1) \) and \( a_2(z_1, z_2) \) that guarantee the exponential stability of EHA position control systems. Substituting these equations into equation (34), the error dynamics can be represented as
\[ \begin{aligned} 
\dot{z}_1 &= z_2 - k_1 z_1 \\
\dot{z}_2 &= z_3 - k_2 z_2 - z_1 \\
\dot{z}_3 &= \dot{f} - k_3 z_3 - z_2 
\end{aligned} \]
(35)

From equation (35), the LCF is defined as
\[ V_4 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} z_3^2 + \frac{1}{2} \dot{f}^2 \]
(36)

where \( \gamma \) is a positive constant.
The derivative of equation (36) can be described as

\[
\dot{V}_4 = z_1 \ddot{z}_1 + z_2 \ddot{z}_2 + z_3 \ddot{z}_3 + \frac{1}{\gamma}f^+ \dot{f}
\]

\[
= -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 + \dot{f} \left( z_3 - \frac{1}{\gamma} \dot{f} \right) \leq 0
\] (37)

From equation (37), an estimation rule to guarantee the system stability can be obtained as

\[
\dot{\hat{f}} = \dot{f} - \gamma z_3
\] (38)

Equation (38) uses the information of \( z_3 \), which depends on the information of \( z_1 \) and \( z_2 \). Therefore, equation (38) closely relates to \( z_1 \) and \( z_2 \), which guarantee the stability of BSC systems.

However, equation (38) cannot be used to the estimation rule because the value of \( f \) is unknown. On the other hand, if \( f \) is assumed as a lumped uncertainty, system uncertainty \( f \) can be estimated by \( \dot{\hat{f}} = -\gamma z_3 \). However, since the value of \( f \) for the EHA system is changed according to the operating condition, it cannot be assumed as the lumped uncertainty. Therefore, to obtain the value of \( f \), the classical discrete disturbance observer scheme was used. Assuming that the sampling rate of the control loop is very fast, the classical discrete disturbance observer expressed by the difference equation is induced from equation (31) as follows

\[
f(k-1) = bu(k) - \dot{\hat{a}}_2(k) + k_3 z_3(k) + z_2(k)
\] (39)

To analyse the stability of the proposed control scheme, equation (38) is substituted into equation (37). Then

\[
\dot{V}_4 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 = -z^T K z < 0
\] (40)

where \( K \) is the diagonal matrix of \( k_1 \), \( k_2 \) and \( k_3 \),

\[ z = [z_1 \ z_2 \ z_3]^T, \text{ and } \dot{V}_4 = 0 \text{ if } z = 0.
\]

If \( z \) is bounded, equation (40) can be defined as

\[
\Phi(t) = z^T K z \geq 0
\] (41)

Integrating equation (41) from 0 to \( t \), the following result can be obtained

\[
\int_0^t \Phi(t) \, dt = V_4[z(0), \dot{f}(0)] - V_4(t)
\] (42)

Applying Barbalat’s Lemma [12] to equation (42), it is possible to deduce that \( \Phi(t) \to 0 \) as \( t \to \infty \). Therefore

\[
\lim_{t \to \infty} \int_0^t \Phi(t) \, dt \leq V_4[z(0), \dot{f}(0)] < \infty
\] (43)

4 COMPUTER SIMULATION

In order to evaluate the validity of the proposed control scheme for EHA position control systems, a sinusoidal reference input was considered as follows

\[
x_d = \sin (0.25 \pi t) + \sin (0.05 \pi t) \text{ [cm]}
\] (44)

This sinusoidal reference input is suitable for evaluating the tracking performance and the robustness of EHA position control systems because it reflects the various changes in the magnitudes of the velocity and position of the piston. Table 1 shows the system parameters of the EHA system which are used to computer simulation. Figure 2 shows the block diagram of the EHA position control system.

Figure 3 shows the simulated tracking errors of the BSC and ABSC systems for the sinusoidal reference. This result shows that the ABSC system has better tracking performance than the BSC system and has error repeatability precision of higher reliability than the BSC system. In addition, in both position and

---

### Table 1: System parameters of the EHA system

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_0 )</td>
<td>Initial volume of the chamber</td>
<td>( 3.712 \times 10^{-4} \text{ m}^3 )</td>
</tr>
<tr>
<td>( A )</td>
<td>Pressure area of the piston</td>
<td>( 5.58 \times 10^{-3} \text{ m}^2 )</td>
</tr>
<tr>
<td>( M )</td>
<td>Piston mass</td>
<td>5 kg</td>
</tr>
<tr>
<td>( \beta_c )</td>
<td>Effective bulk modulus</td>
<td>( 1.7 \times 10^6 \text{ MPa} )</td>
</tr>
<tr>
<td>( L_t )</td>
<td>Leakage factor of the pump</td>
<td>( 3.16 \times 10^{-16} \text{ m}^3/\text{Pa} )</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>Bristles stiffness coefficient</td>
<td>( 5.77 \times 10^6 \text{ N/m} )</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>Bristles damping coefficient</td>
<td>( 2.28 \times 10^4 \text{ N/s/m} )</td>
</tr>
<tr>
<td>( \omega_{op \ max} )</td>
<td>Maximum speed of the motor</td>
<td>178 rad/s</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Volumetric capacity of the pump</td>
<td>( 1.591 \times 10^{-6} \text{ m}^3/\text{rad} )</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>Coulomb friction coefficient</td>
<td>370 N</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>Static friction</td>
<td>217 N</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>Viscous friction coefficient</td>
<td>2318 N/m/s</td>
</tr>
<tr>
<td>( \nu_{sv} )</td>
<td>Stribeck velocity</td>
<td>0.032 m/s</td>
</tr>
</tbody>
</table>

---

Fig. 2 Block diagram of the EHA position control system
control schemes relatively large tracking errors occur at the nearly zero velocity regions. This is attributable to the effect of dynamic friction characteristics, which produce an instantaneous large force at the nearly zero velocity regions. For the transient response region of the initial operation of EHA position control systems, the ABSC system with the estimator for system uncertainties yields approximately 40 per cent improvement compared with the BSC system without the estimator because the $f$ in equation (31) including system uncertainties is estimated by equation (43), as shown in Fig. 4. Figure 4 shows the estimated value $\hat{f}$ for the system uncertainties of EHA systems obtained by the proposed adaptive rule. The estimated value plays a role in the consideration of non-linearity and uncertainties included in EHA systems.

Figure 5 shows the tracking errors of the BSC and ABSC systems having perturbations of the system parameters such as the Coulomb friction, viscous friction, and pump leakage coefficient in the EHA system for the sinusoidal reference input. It was assumed that the system parameters have a perturbation of $\pm 50$ per cent. From Fig. 5, it was found that the perturbations of the system parameters of the EHA system are closely related with the tracking performance of the EHA system. Table 2 shows the tracking root mean square (RMS) errors of the BSC and ABSC systems according to the perturbation of the system parameters. The variations of the tracking RMS errors due to the 50 per cent perturbation of the system parameters for the BSC and ABSC systems are 17.6 per cent and 3.02 per cent respectively. These results show that the proposed position control scheme has the desired robustness to system uncertainties such as the perturbation of the viscous

**Table 2** Tracking RMS errors of the BSC and ABSC systems according to the perturbations of the system parameters

<table>
<thead>
<tr>
<th>Control scheme</th>
<th>Perturbation ratio</th>
<th>RMS value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSC</td>
<td>0 per cent</td>
<td>1.878 mm</td>
</tr>
<tr>
<td></td>
<td>$\pm 50$ per cent</td>
<td>2.209 mm</td>
</tr>
<tr>
<td>ABSC</td>
<td>0 per cent</td>
<td>0.265 mm</td>
</tr>
<tr>
<td></td>
<td>$\pm 50$ per cent</td>
<td>0.273 mm</td>
</tr>
</tbody>
</table>
friction, Coulomb friction, and pump leakage coefficient.

5 EXPERIMENTAL RESULTS AND DISCUSSION

Figure 6 shows the experimental set-up of the EHA system. To evaluate the effectiveness of the proposed control system, the PCM-3350 (AMD Geode processor, 300MHz) was used. The control algorithms were programmed by Turbo-C++ language on MS-DOS, in order directly to handle the PCM-3718 as a data acquisition board. The PCM-3718 is a fully multifunctional card with PC/104 interface. In addition, to measure the position of the piston, a linear variable differential transformer (LVDT) sensor was used. The sampling rate was set to 1 kHz.

Figure 7 shows the experimental tracking errors of the BSC and ABSC systems for the sinusoidal reference input, which was used in the computer simulation. The tracking error of the BSC system is relatively large when the direction of the piston is changed because the BSC system cannot compensate the friction of the EHA system. In addition, the tracking error of the BSC varies according to the direction of the piston because of the system uncertainties of the EHA system. However, the ABSC system has better tracking performance than the BSC system because the ABSC system can effectively compensate the system uncertainties as well as the non-linear friction effects by using the estimated value $f$, which is shown in Fig. 8. Figure 9 shows the
speed of the motor as the control input for the sinusoidal reference input.

Figure 10 shows the tracking errors of the BSC and ABSC systems for the square wave type reference input. The characteristics of the transient responses of the BSC and the ABSC systems are almost the same. In the BSC system, however, steady-state error occurs relatively large in the backwards direction. This shows that the BSC system cannot compensate the system uncertainties of the EHA system. But it can be shown that the ABSC system can effectively compensate the system uncertainties regardless of the piston direction. Figure 11 shows the estimated value $f$ for the system uncertainties of the ABSC system for the square wave type reference input. The estimated value $f$ for the system uncertainties makes the desired tracking performance and robustness to the EHA system with system uncertainties. Figure 12 shows the speed of the motor as the control input for the square wave type reference input.

Table 3 shows the tracking RMS errors of the BSC and ABSC systems for the sinusoidal reference input and the square wave type reference input at steady state. From Table 3, it was found that using the ABSC system instead of the BSC system yields about five times improvement in the tracking performance of the EHA position control system.

6 CONCLUSION

A robust position control of EHA systems was proposed by using the ABSC scheme, which has robustness to system uncertainties such as the perturbation of viscous friction, Coulomb friction, and pump leakage coefficient. First, a stable BSC system based on the EHA system dynamics was derived. However, the BSC scheme had a drawback: it could not consider system uncertainties. To overcome the drawback of the BSC scheme, the ABSC scheme was proposed having error equations for the velocity, acceleration and jerk of the piston respectively, which were induced by the BSC scheme. To

![Fig. 10](image-url) Tracking errors of the BSC and ABSC systems for the square wave type reference input

![Fig. 11](image-url) Estimated value for the system uncertainties of the ABSC system for the square wave type reference input

![Fig. 12](image-url) Speed of the motor as the control input for the square wave type reference input

<table>
<thead>
<tr>
<th>Control system</th>
<th>Sinusoidal reference input</th>
<th>Square wave type reference input at steady state</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSC</td>
<td>1.762 mm</td>
<td>0.395 mm</td>
</tr>
<tr>
<td>ABSC</td>
<td>0.309 mm</td>
<td>0.114 mm</td>
</tr>
</tbody>
</table>
evaluate the performance and robustness of the proposed EHA position control system, BSC and ABSC schemes were implemented in a computer simulation and experiment. It was found that the ABSC scheme can yield the desired tracking performance and the robustness to system uncertainties.

ACKNOWLEDGEMENTS

This research was financially supported by the Ministry of Education, Science Technology (MEST) and Korea Industrial Technology Foundation (KOTEF) through the Human Resource Training Project for Regional Innovation

© Authors 2010

REFERENCES


12 Krstic, M., Kanellakopoulos, I., and Kokotovic, P. Nonlinear and adaptive control design, 1995 (Wiley Interscience, New York, USA).